Engineering Mathematics II (10320MS 201201), 2015S

Class Time: T3T4/R3R4 (200 minutes)

Class Room: 台達 B105

Instructor: Ta-Jen Yen (嚴大任)
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Textbooks: Advanced Engineering Mathematics (2nd edition), *Michael D. Greenberg*, 滄海書局

Grading: Quizzes (x3, 25%)
Midterms (x2, 50%)
Final (25%)

TAs: Olivia (江叡涵): olivia30127@gmail.com
TA Hours: 7:00-8:30 pm, every other Thursday (TBA)

Readers: Not listed here

Final (25%)

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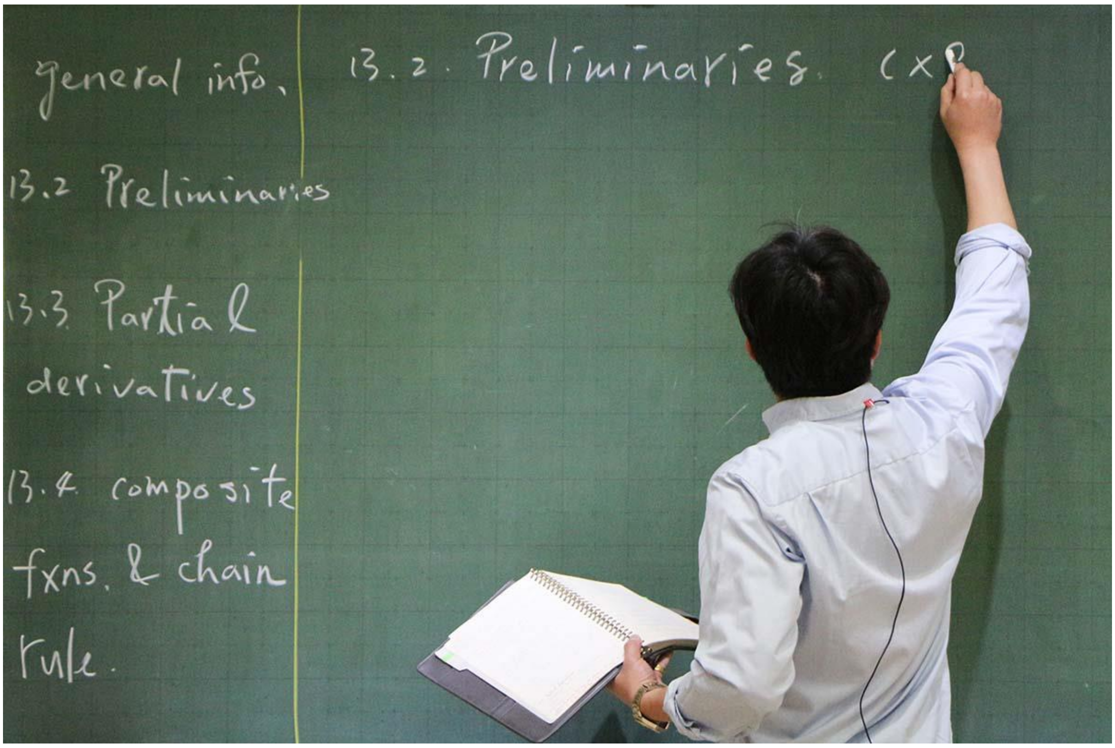
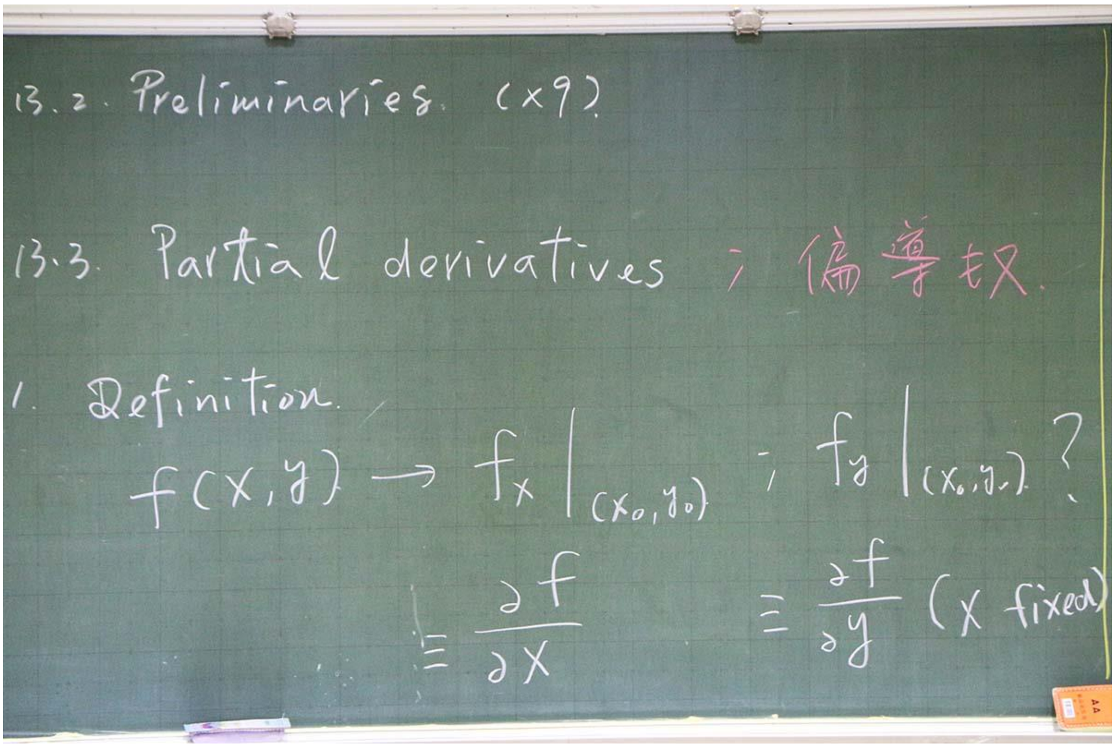
Content:

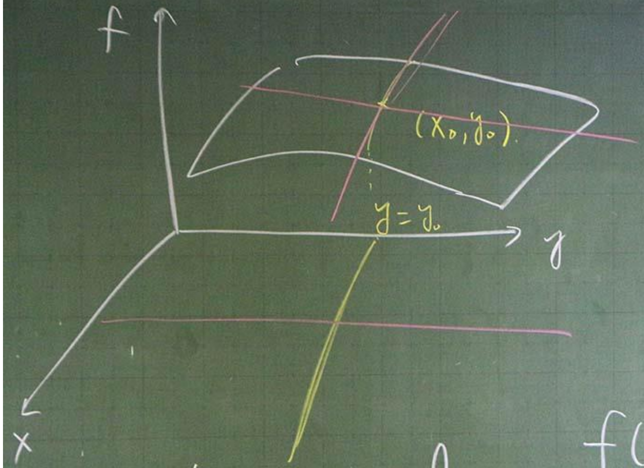
The subject of ***Engineering Mathematics*** is comprised of five parts within two semesters:

1. Ordinary Differential Equations
2. Linear Algebra
3. ***Multivariable Calculus and Field Theory***
4. ***Fourier Methods and Partial Differential Equations***
5. ***Complex Variable Theory***

In Engineering Mathematics II, we focus on "***Multivariable Calculus and Field Theory***", "***Fourier Methods and Partial Differential Equations***", and "***Complex Variable Theory***" to cover the rationale and of particular the engineering applications. The detailed schedule of this semester is listed below:


general info.
13.2 Preliminaries
13.3 Partial derivatives
13.4 composite fxns. & chain rule.
13.2. Preliminaries.


13.2. Preliminaries. (x9)
13.3. Partial derivatives ; 偏導數.
1. Definition.
$f(x,y) \to f_x \big|_{(x_0,y_0)}$; $f_y \big|_{(x_0,y_0)}$?
$\equiv \frac{\partial f}{\partial x}$
$\equiv \frac{\partial f}{\partial y}$ (x fixed)



$$f_x\Big|_{(x_0,y_0)} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}\Bigg|_{(x_0,y_0)}$$

$$f_{xy} = \partial\left(\frac{\partial f}{\partial x}\right)/\partial y = \frac{\partial^2 f}{\partial y\, \partial x} \quad \#$$

<EX.1> Let $f(x,y) = x\sin(xy^2)$, find f_x, f_y, f_{xy}, f_{yx}.

Sol

$$f_x = \sin(xy^2) + x\cos(xy^2)\cdot y^2$$

$$= \sin(xy^2) + xy^2\cos(xy^2)$$

$$f_y = x\cos(xy^2)\cdot(2xy) = 2x^2y\cos(xy^2)$$

$$f_{xy} = \cos(xy^2)(2xy) + 2xy\cos(xy^2) - xy^2\sin(xy^2)(2xy) = -2x^2y^3\sin(xy^2) + 4xy\cos(xy^2)$$

$$f_{yx} = 4xy\cos(xy^2) - 2x^2y\sin(xy^2)\cdot y^2$$

$$= 4xy\cos(xy^2) - 2x^2y^3\sin(xy^2)$$

$$f_{xy} = f_{yx}$$

#

<Ex.2>

$$f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$$

find f_{xy} & f_{yx} ? $\rightarrow f_{xy} \neq f_{yx}$

<Theorem> Equality of derivatives.

$f(x,y) \rightarrow$ if f_x, f_y, f_{xy}, f_{yx} are all "continuous" in some neighborhood of (x_0, y_0)

, then $f_{xy} = f_{yx}$

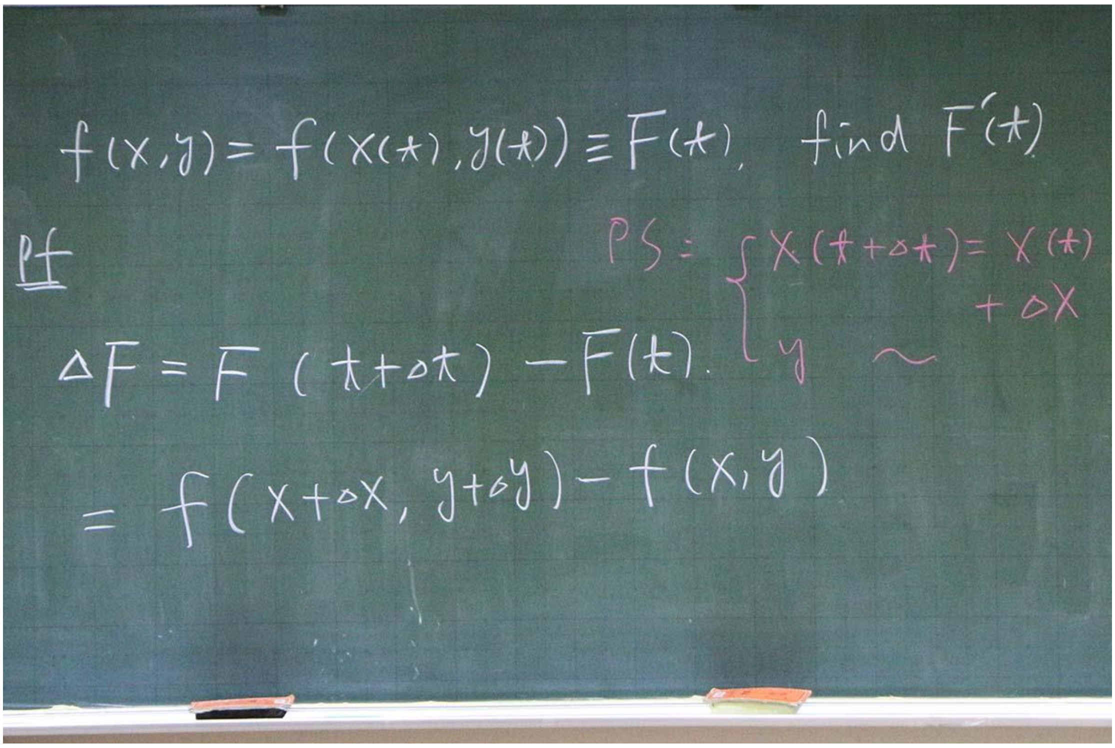
13.4 Composite fxns. & chain rule.

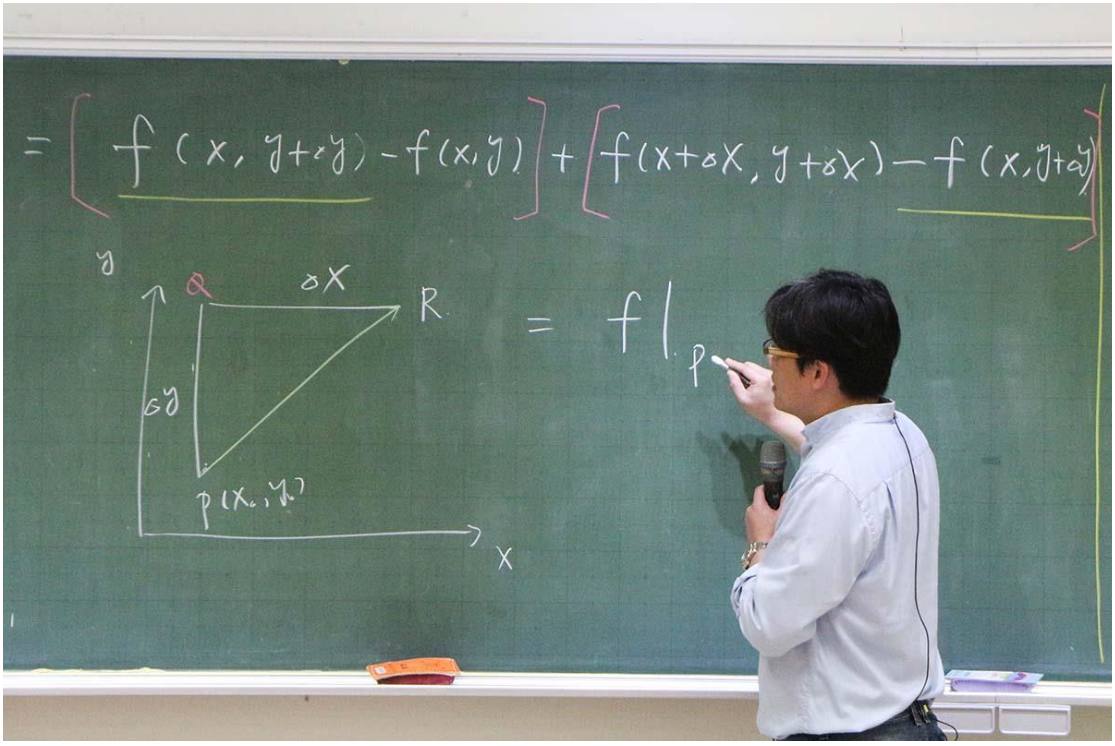
1. single-variable fxns.

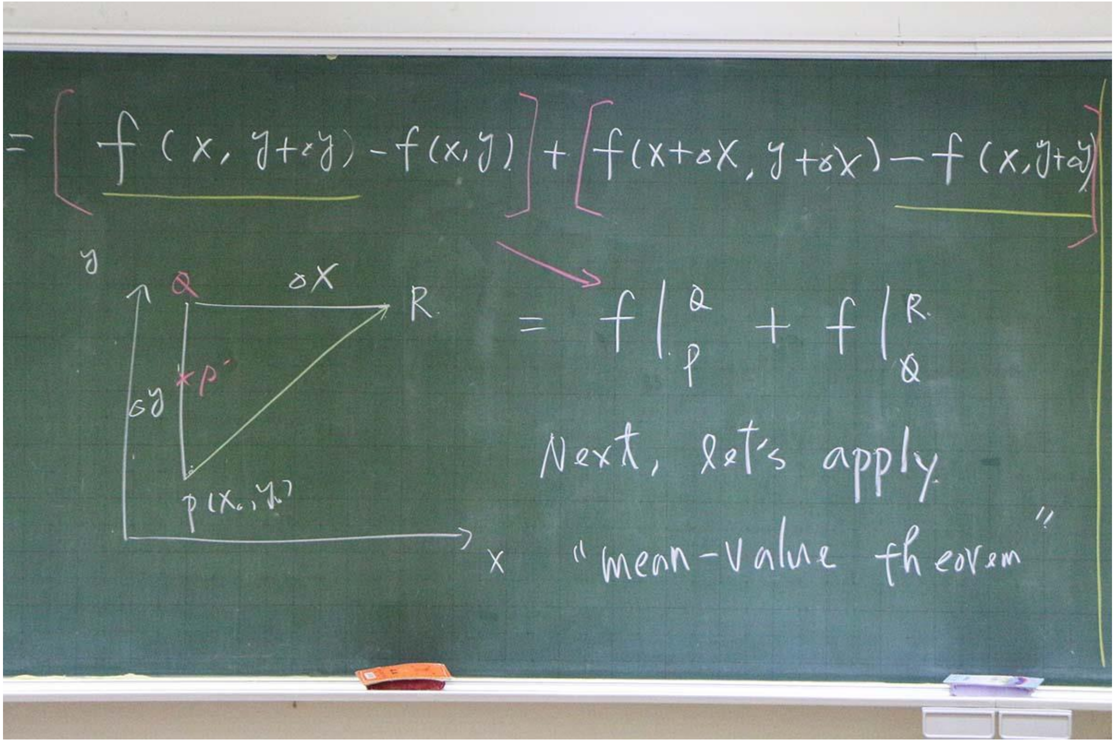
$$f(x) = f(x(t)) \equiv F(t)$$ composite fxn.

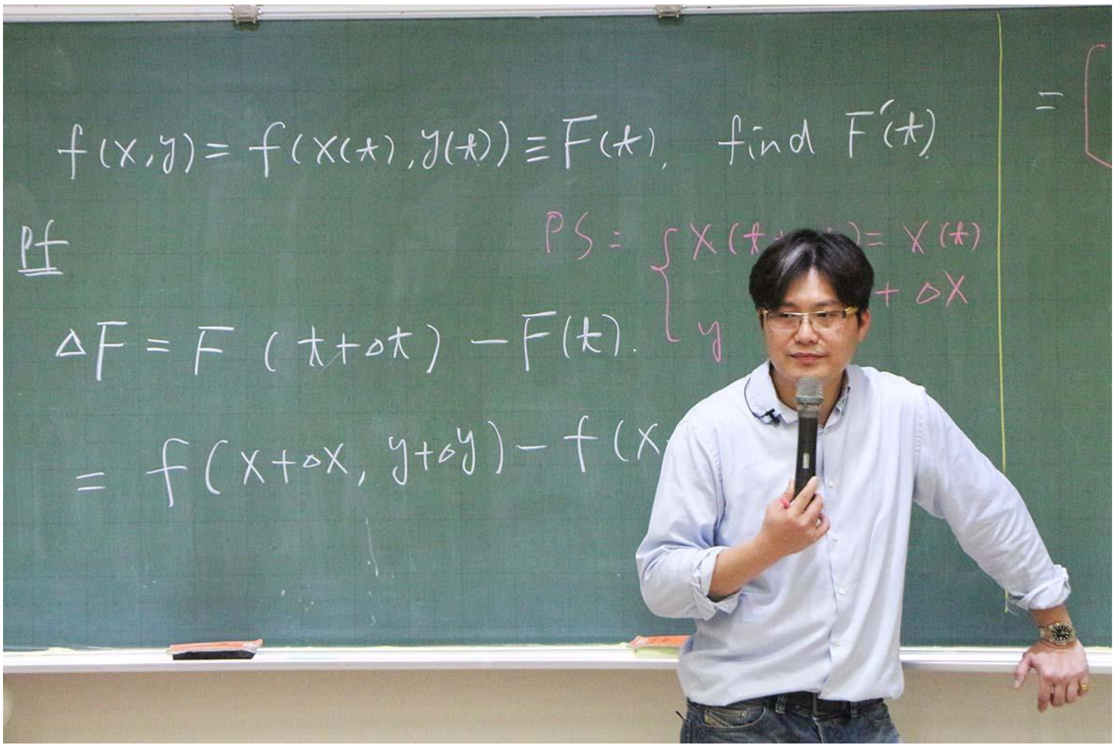
"prime"

$$\frac{dF(t)}{dt} = F'(t) = \underbrace{\frac{df(x)}{dx} \cdot \frac{dx}{dt}}_{\text{chain rule!}}$$


f(x,y) = f(x(t), y(t)) ≡ F(t), find F'(t)
Pf
PS: x(t+Δt) = x(t) + Δx
y ~
ΔF = F(t+Δt) − F(t)
= f(x+Δx, y+Δy) − f(x,y)


= [f(x, y+Δy) − f(x,y)] + [f(x+Δx, y+Δx) − f(x, y+Δy)]
y
Q
Δx
R
Δy
P(x, y)
x
= f |P


= [f(x, y+Δy) − f(x,y)] + [f(x+Δx, y+Δx) − f(x, y+Δy)]
= f|_P^Q + f|_Q^R
Next, let's apply "mean-value theorem"
Q
R
Δx
Δy
P
p(x, y)
x
y


f(x,y) = f(x(t), y(t)) ≡ F(t), find F'(t)
Pf
ΔF = F(t+Δt) − F(t)
= f(x+Δx, y+Δy) − f(x
PS:
+ Δx

$$\Delta F = \frac{\partial f}{\partial y}\Big|_{P'} \Delta y + \frac{\partial f}{\partial x}\Big|_{Q'} \Delta x \quad \text{——— (1)}$$

As $\Delta t \to 0$, $\Delta x \to 0$ and $\Delta y \to 0$

Then, $P' \to P$, $Q' \to P$.

$$\lim_{\Delta t \to 0} \frac{\Delta F}{\Delta t} = \lim_{\Delta t \to 0} \left(f_y \frac{\Delta y}{\Delta t} + f_x \frac{\Delta x}{\Delta t} \right)$$

$$= f_x x' + f_y y' \quad \#$$

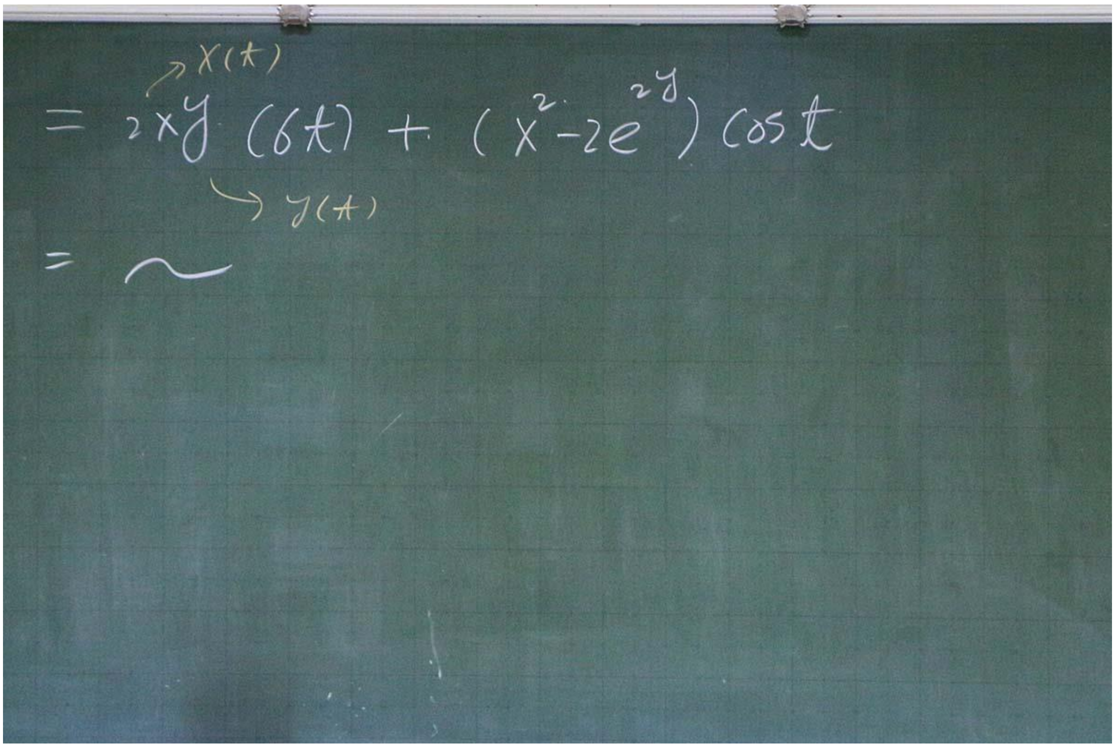
composite fxn

<Ex.1>

$r(x,y) = x^2 y - e^{2y} \equiv R(t)$, where

$x(t) = 3t^2$, $y(t) = \sin t$. Find $R'(t)$.

Sol. $R'(t) = \dfrac{\partial r}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial r}{\partial y}\dfrac{dy}{dt}$



$x(t)$
$= 2xy\,(6t) + (x^2 - 2e^{2y})\cos t$
$y(t)$
$= \sim$